Spinnability Simulation of Viscoelastic Fluid

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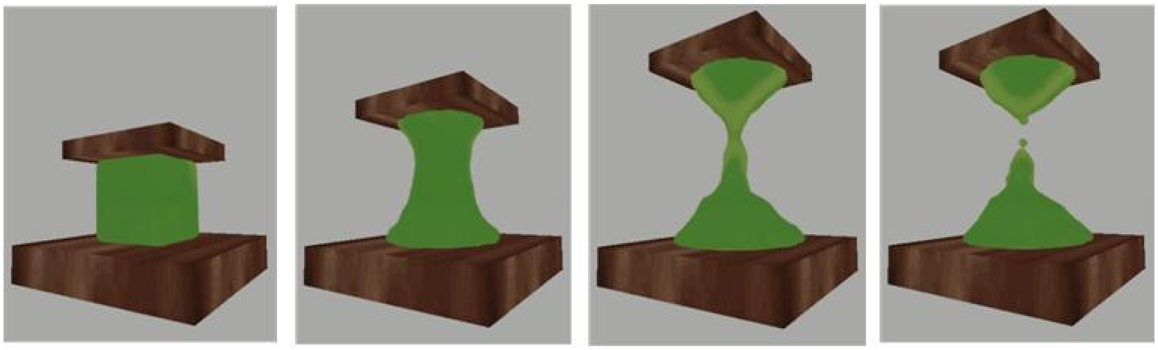

Figure 1: *A sequence of images showing the spinnability property of viscoelastic fluid. (a) The initial state, where the middle part is viscoelastic fluid and the both end are solid. (b) The fluid is being lifted as the top solid is moving. (c) The fluid is stretched just before its string is broken. (d) The state just after the string is broken.*

1 Introduction

One of the most challenging issues of computer graphics is to represent the behavior of fluid. Visualizing the fluid behavior requires to solve Navier-Stokes equations, which take huge amount of time so that some researches use many super computers for the simulation, and others utilize the GPU performance. The common fluid is Newtonian that can be described by a single constant value of viscosity, and there are many researches related to Newtonian. On the other hand, there is another type of fluid called non-Newtonian that cannot be described easily, and one of non-Newtonians is viscoelactic fluid. Viscoelastic fluid has the characteristics of both viscosity of fluid and elasticity of solid, and it is difficult to represent the behavior of viscoelastic fluid. [Goktekin et al. 2004] represented the behavior of viscoelastic fluid. His technique is based on Eulerian methods and added elastic terms to Navier-stokes equations, which govern fluid behavior. [Clavet et al. 2005] used particle method for representing fluid behavior. Particle method can represent fine behavior of the fluid such as rain drops, fountains, clay manipulation. Their researches could visualize many types of behavior of viscoelastic fluid, however, they cannot represent the spinnability, which has three characteristics: 1) it stretches very thin as if it is a string, 2) the radius is getting smaller gradually from the both ends and the center part has the least radius, and 3) it shrinks rapidly as if it is a rubber.

In this paper, we describe a method of animating the spinnability behavior of viscoelastic fluid. The method calculates Cauchy's equations of motion, and constitutive equations of viscoelastic fluid and solvent by using MPS (Moving Particle Semi-implicit) method as the governing equation of motion. In addition, surface tension is integrated for representing the property of spinnability.

2 Method

The method uses the following equations.

Equation of continuity:

$$\frac{d\rho}{dt} = 0 \tag{1}$$

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Cauchy's equations of motion with surface tension:

$$\frac{d\boldsymbol{v}}{dt} = \frac{1}{\rho}\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{G} + \frac{1}{\rho}\kappa\gamma\delta\boldsymbol{n} \tag{2}$$

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau} \tag{3}$$

Constitutive equations of viscoelastic fluid and solvent:

$$\boldsymbol{\tau} = \boldsymbol{\tau_s} + \boldsymbol{\tau_p} \tag{4}$$

$$\boldsymbol{\tau_s} = 2\eta_s\boldsymbol{D} \tag{5}$$

Giesekus model:

$$\boldsymbol{\tau_p} + \lambda \overset{\nabla}{\boldsymbol{\tau_p}} + \alpha\frac{\lambda}{\eta_p}\boldsymbol{\tau_p} \cdot \boldsymbol{\tau_p} = 2\eta_p\boldsymbol{D} \tag{6}$$

$$\overset{\nabla}{\boldsymbol{\tau_p}} = \frac{d\boldsymbol{\tau_p}}{dt} - \boldsymbol{L} \cdot \boldsymbol{\tau_p} - \boldsymbol{\tau_p} \cdot \boldsymbol{L}^t \tag{7}$$

$$\boldsymbol{L} = \nabla\boldsymbol{v}, \boldsymbol{D} = \frac{1}{2}(\boldsymbol{L} + \boldsymbol{L}^t) \tag{8}$$

where, ρ is density, t is time, $\boldsymbol{v}$ is velocity, $\boldsymbol{\sigma}$ is stress tensor, $\boldsymbol{G}$ is gravity, κ is curvature, γ is surface tension coefficient, δ is delta function, $\boldsymbol{n}$ is normal vector of surface, p is pressure, $\boldsymbol{I}$ is unit matrix, $\boldsymbol{\tau_s}$ is solvent stress, $\boldsymbol{\tau_p}$ is viscoelastic stress, η_s is solvent viscosity, $\boldsymbol{D}$ is velocity tensor by deformation, λ is relaxation time, α is constant, and η_p is viscoelastic viscosity.

Finally, we could simulate the spinnability of viscoelastic fluid with the proposed method. Analysis time by MPS was 155 ms per step, polygon generation time by marching cube method was 280 ms, and rendering time with OpenGL was 16 ms for the model composed of 2,744 fluid particles and 5,007 solid particles.

References

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GOKTEKIN, T. G., BARGTEIL, A. W., AND O'BRIEN, J. F. 2004. A method for animating viscoelastic fluids. *Proceedings of the 2004 SIGGRAPH*, 463–468.